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Math 131 - Spring 2025 - Common Final Exam

Print name: Smarty Pants version A

Section number: 000 **Instructor's name:** J. Del Greco

Directions:

- This exam has 14 questions. Please check that your exam is complete, but otherwise keep this page closed until the start of the exam is called.
- Fill in your name, and your instructor's name.
- It will be graded out of 92 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a calculator which cannot connect to the internet. The use of any notes or electronic devices other than a calculator is prohibited.
- A formula sheet has been provided with this exam. You may not refer to any other notes during the exam.

Good luck!

Question:	1	2	3	4	5	6	7	8
Points:	6	3	6	7	12	11	10	6
Score:								

Question:	9	10	11	12	13	14		Total
Points:	5	6	4	4	6	6		92
Score:								

1. Sand is continuously falling onto a pile. The height of the pile, h (in meters), is measured at various times, t (in hours).

t	3.20	3.40	3.60	3.80	4.00
$h(t)$	4.73	5.63	5.91	6.12	6.22

(a) (3 points) What is the **average rate of change** in the height of the sand pile from $t = 3.2$ to $t = 4.0$? You must include units in your answer and round to two decimal places.

$$\frac{6.22 - 4.73}{0.8} = 1.86 \text{ m/hr}$$

(+1) (+1) (+1)

(b) (3 points) Estimate the **instantaneous rate of change** of the pile's height at $t = 3.6$. Use an estimate of left and right intervals. You must include units in your answer and round to two decimal places.

$$\frac{6.12 - 5.91}{.2} = 1.05$$

(+1)

$$\frac{5.63 - 5.91}{-.2} = 1.4$$

(+1)

(+1) $$\frac{1.05 + 1.4}{2} = \frac{2.45}{2} = 1.225 \text{ m/hr}$$

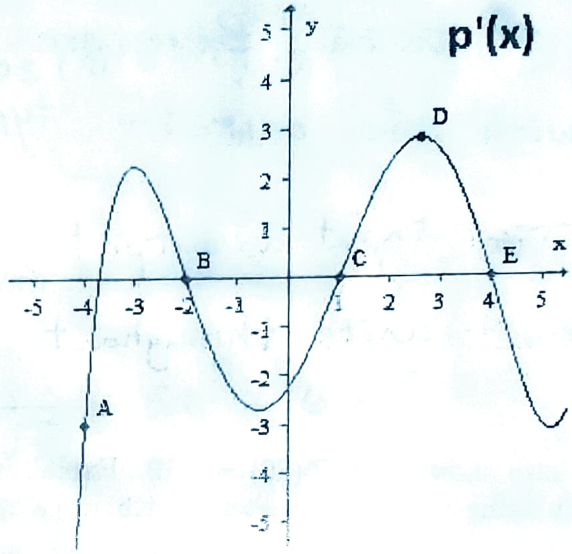
2. (3 points) Approximate $s'(2)$ using $h = 0.01$ if $s(x) = \sin(\ln(x + 4))$. (Be sure that your calculator is set in radians and round your answer to three decimal places.)

$$\frac{s(2.01) - s(2)}{0.01} = -0.037$$

(+1) (+1)

(+1) for use of $x = 2.01$

3. (6 points) Use the graph of $p'(x)$ below to answer questions about $p(x)$, $p'(x)$, and $p''(x)$. Circle all points that apply for each question. Choose NA if none of the points apply. CAUTION: **The graph below is that of $p'(x)$ and not $p(x)$.**



(a) At which point(s), if any, is $p(x)$ increasing?

A B C D E NA

(b) At which point(s), if any, is $p'(x)$ increasing?

A B C D E NA

(c) At which point(s), if any, is $p''(x)$ negative?

A B C D E NA

(d) Which point(s), if any, are critical points of $p(x)$?

A B C D E NA

+1 each part

(e) Which point(s), if any, are local minimums of $p(x)$?

A B C D E NA

(f) Which point(s), if any, are local maximums of $p(x)$?

A B C D E NA

4. Let P(t) represent the number of people currently infected with a new type of flu virus in the state of Washington, where $t = 1$ is January 1, 2025.

(a) (2 points) Data shows that $P(30) = 487$. Explain the meaning of this statement about the new flu using a complete sentence without calculus terms.

On January 30, 2025, there are 487 people infected with the new flu type.

+1 for correct input and output

+1 for correct units throughout

(b) (2 points) Data also shows that $P'(30) = -19$. Explain the meaning of this statement about the new flu using a complete sentence without calculus terms.

On January 30, 2025, the new cases of the flu are decreasing at approximately 19 cases/day.

+1 for correct input and output

+1 for correct units throughout

(c) (3 points) Using the values in the previous parts, calculate an estimate for $P(28)$. Explain the meaning of this statement about the new flu using a complete sentence without calculus terms.

+1

$$
\begin{aligned}
P(28) &\cong P'(30)(-2) + P(30) \\
&= (-19)(-2) + 487 \\
&= 38 + 487 \\
&= \underline{\underline{525}} \text{ cases}
\end{aligned}
$$

+1

+1

From January 28 to January 30, the flu cases decreased by 38 cases (approximately).

5. Find the requested derivatives. You are not required to simplify your final answer.

a. $f'(t)$ for $f(t) = sin(\pi t + 1) - \pi$ (3 points)

$$f'(t) = \cos(\pi t + 1) \cdot \pi$$

+2

+1 for chain rule use

b. $g'(x)$ for $g(x) = \sqrt{x} + x^5 - \frac{2}{x^3}$ (3 points)

$$g'(t) = \frac{1}{2\sqrt{x}} + 5x^4 + 6x^{-4}$$

+1 for each term

c. $r'(x)$ for $r(x) = e^{\cos(3x-1)}$ (3 points)

$$r'(x) = e^{\cos(3x-1)} \cdot (-\sin(3x-1)) \cdot 3$$

+1 for chain rule use

+2 for correct answer

d. $m'(t)$ for $m(t) = (3^t)(\ln(t))$ (3 points)

$$m'(t) = 3^t\left(\frac{1}{t}\right) + \ln(t) \cdot 3^t \ln(3)$$

+1 for product rule use

+2 for correct answer

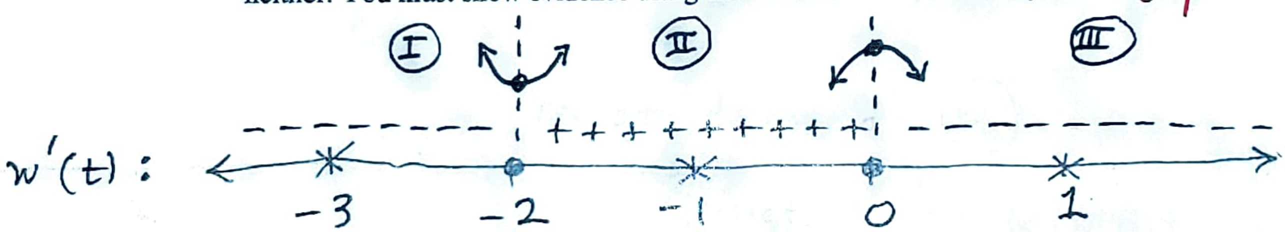
6. For the questions below, use the function $w(t) = 13 - 3t^2 - t^3$ on $-4 \le t \le 0$.

a. Find all critical points of $w(t)$ in its domain. (4 points)

$$w'(t) = -6t - 3t^2 = -3t(2+t) = 0 \Rightarrow t = 0, -2$$

(+2) (+2)

b. Classify each of the critical points from part (a) as a local maximum, local minimum, or neither. You must show evidence using derivative tests to receive any credit. (3 points)



x = test point

I: $-3(-3)(2-3) < 0$

II: $-3(-1)(2-1) > 0$

III: $-3(1)(2+1) < 0$

$\boxed{\begin{array}{l} x = 0 \text{ is a local max} \\ x = -2 \text{ is a local min} \end{array}}$

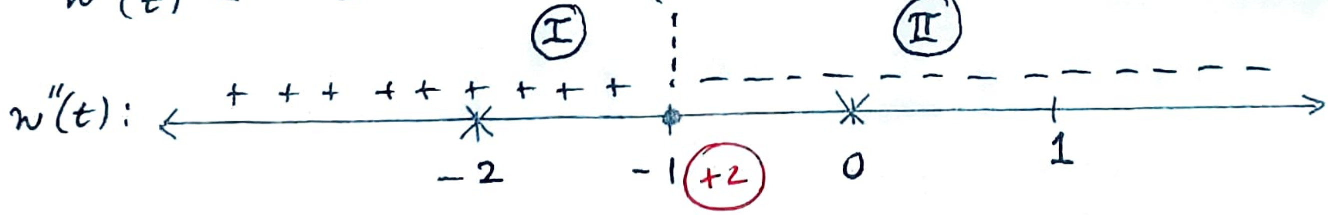
(+1) for evidence of either derivative test

(+1) for each correct classification

c. Find any inflection points of $w(t)$. Use must calculus to confirm that the point are actually inflection points. You show all work and reasoning to receive any credit. (4 points)

$$w''(t) = -6 - 6t = -6(1+t) = 0 \Rightarrow t = -1$$

(+1) (+1)



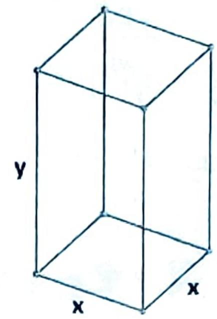
x = test point

I: $-6(1-2) > 0$

II: $-6(1+0) < 0$

$\boxed{x = -1 \text{ is an inflection point}}$

7. A company is designing a storage unit in the shape of a rectangular box with a square top and bottom (see the diagram below). The box must hold a volume of 27,000 cubic meters, and designers want to minimize the box's surface area to lower construction costs.



(a) (2 points) Write a formula for the surface area of the box. Assume the width of the square base is x and the height of the box is y.

$$S(x, y) = 2x^2 + 4xy$$

(+1) sum of □ sides' areas

(+1) correct expression

(b) (2 points) Write a formula for the total volume of the box. Assume the width of the square base is x and the height of the box is y.

$$V(x,y) = x^2 y$$

(+1) use of box volume formula

(+1) correct expression

(c) (2 points) Write a formula for the box's surface area in terms of a single variable.

$$x^2 y = 27{,}000 \Rightarrow y = \frac{27{,}000}{x^2} \quad (+1)$$

$$\therefore S(x) = 2x^2 + 4x\left(\frac{27{,}000}{x^2}\right) \quad (+1)$$

$$= 2x^2 + \frac{108{,}000}{x}$$

(d) (4 points) **Use calculus** to find the minimum required (i.e. the global minimum) surface area of the box. You must show all calculations and formulas used to receive credit.

$$S'(x) = 4x - \frac{108{,}000}{x^2} = 0 \Rightarrow \frac{108{,}000}{x^2} = 4x$$

(+1) (+1)

$$\Rightarrow 4x^3 = 108{,}000$$

$$\Rightarrow x^3 = 27{,}000$$

$$\Rightarrow x = 30$$ is a global minimum (+1)

S(x)

$S'(x)$: − − − − − − − − − − (0) − − − (30) + + + + + + + +

$$S(30) = 2(30)^2 + \frac{108000}{30} = 5{,}400 \text{ m}^2$$

(+1)

8. Maria runs a chemical reagent business. She calculates her monthly revenue using the function $R(q) = 2100 \cdot \ln(10q + 20)$ and her monthly costs using the function $C(q) = 5000 + 30q$, where q stands for the number of chemical containers produced.

   (a) (2 points) Write a formula for the profit Maria earns every month.

   $P(q) = 2100 \ln(10q + 20) - 5000 - 30q$ +2

   (b) (3 points) Use calculus to determine the quantity of containers (q) she needs to sell in a month to maximize her profit.

   $P'(q) = \dfrac{21000}{10q+20} - 30 = 0 \Rightarrow 21{,}000 = 300q + 600$ +2

   $\Rightarrow 20{,}400 = 300q$

   $\Rightarrow q = \underline{\underline{68}}$ +2

   (c) (1 point) Calculate the profit she expects to make in a month by selling the quantity determined in part (b). Round your answer to the nearest dollar.

   $P(68) = 2100 \ln(680 + 20) - 5{,}000 - 2{,}040$

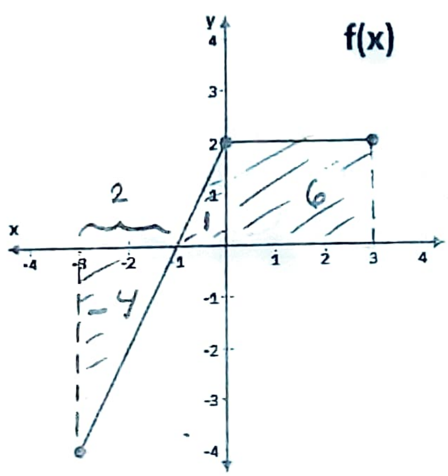
   $= 2100 \ln(700) - 7{,}040$

   $\cong \underline{\underline{\$ 6{,}717}}$ +1

9. (5 points) Using the figure below, find $\int_{-3}^{3} f(x)\, dx$. Show all of your work and calculations.

+1 sum of areas

+1 negative value over $-3 \leq x \leq -1$



$$\int_{-3}^{3} f(x)\, dx = -\frac{1}{2}(2)(4) + \frac{1}{2}(1)(2) + (3)(2)$$

$$= -4 + 1 + 6 = \underline{\underline{3}}$$ +1

10. A polluting chemical is being removed from a large pool at a rate of $r(t)$ (in liters per hour). The data in the table below gives the rate at varying times since the removal process began.

t	0	3	6	9	12
$r(t)$	8.7	7.2	6.0	5.2	4.7

(a) (3 points) Use the table to estimate $\int_0^{12} r(t)\,dt$. Use an average of the upper and lower estimates.

$\Delta t = 3$

$R(4) = (3)(7.2 + 6.0 + 5.2 + 4.7) = (3)(13.2 + 9.9) = 3(23.1) = 69.3$ +1 +1

$L(4) = (3)(8.7 + 7.2 + 6.0 + 5.2) = (3)(15.9 + 11.2) = 3(27.1) = 81.3$

$\dfrac{69.3 + 81.3}{2} = \dfrac{150.6}{2} = 75.3 \text{ L}$ +1 Approximately 75.3 L were removed.

(b) (2 points) Interpret your answer from part (a) in terms of chemical removal. You must include units.

Approximately 75.3 L of the polluting chemical were removed. +1 units

+1 correct statement

(c) (1 point) If the large pool originally contained 322 liters of the polluting chemical, how much remains after 12 hours?

$322 - 75.3 = 246.7 \text{L}$ (approximately)

+1

11. (4 points) Given that $\int_1^7 f(x)\,dx = 4$, $\int_7^{10} f(x)\,dx = -9$, and $\int_1^{10} g(x)\,dx = -8$, evaluate the expression below.

$$\int_1^{10} (2f(x) + 3g(x))\,dx$$

+1

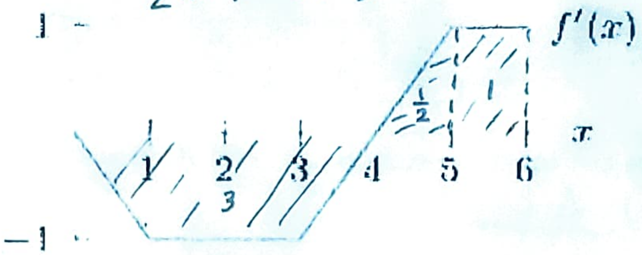
$= 2\int_1^{10} f(x)\,dx + 3\int_1^{10} g(x)\,dx$

$= 2(4 - 9) + 3(-8) = -10 - 24 = -34$

+1 +2

12. (4 points) The graph of $f'(x)$ is shown in the figure below. Given that $f(0) = 50$ find $f(6)$.

$\frac{1}{2}(1)(4+2) = 3$



(+1) Use of areas

(+1) Correct use of FToC

$$3 + \frac{3}{2} \;\; - \frac{3}{2} = \int_0^6 f'(x)\,dx = f(6) - f(0) = f(6) - 50$$

$$\therefore \quad f(6) = -\frac{3}{2} + \frac{100}{2} = \frac{97}{2} = \underline{48.5}$$

(+2)

13. Find the indefinite integrals below.

(a) (3 points) $\int (\sqrt{x} - \frac{8}{x} - 5x)\,dx$

$$= \frac{2}{3}x^{3/2} - 8\ln|x| - \frac{5}{2}x^2 + C$$

(+1) each term

(b) (3 points) $\int (3sin(x) - 24x^3 - 9)\,dx$

$$= -3\cos(x) - 6x^4 - 9x + C$$

(+1) each term

See next page

14. (6 points) For $0 \le x \le 4$ find the **exact area** between $y = 2x^2 - 18$ and the x-axis using the **Fundamental Theorem of Calculus**. You must show all work and reasoning for credit.

$$\int_0^4 2x^2 - 18\,dx = \frac{2}{3}x^3 - 18x\Big|_0^4$$

$$= \frac{2}{3}(4)^3 - 18(4)$$

$$= 4\left(\frac{2}{3}\cdot 16 - 18\right)$$

$$= 4\left(\frac{32}{3} - \frac{54}{3}\right)$$

$$= 4\left(-\frac{22}{3}\right)$$

$$= -\frac{88}{3} = -29.\overline{3}$$

14. (6 points) For $0 \le x \le 4$ find the **exact area** between $y = 2x^2 - 18$ and the x-axis using the **Fundamental Theorem of Calculus**. You must show all work and reasoning for credit.

$$2x^2 - 18 = 0 \Rightarrow 2x^2 = 18 \Rightarrow x^2 = 9$$

$$\Rightarrow x = 3 \quad (+1)$$

(since $0 \le x \le 4$)

Area between x-axis and $y = 2x^2 - 18$ is

$$\int_0^3 -(2x^2 - 18)\,dx + \int_3^4 + (2x^2 - 18)\,dx$$

$$= -\frac{2}{3}x^3 + 18x \Big|_0^3 + \frac{2}{3}x^3 - 18x \Big|_3^4$$

$$= -\frac{2}{3}(27) + 54 + \left(\frac{2}{3}(64) - 72 - \left(\frac{2}{3}(27) - 54\right)\right)$$

$$= -18 + 54 + 42.\overline{6} - 72 - 18 + 54$$

$$= -36 + 108 - 29.\overline{3}$$

$$= 42.\overline{6}$$

$$= \frac{128}{3}$$

(+1)

(+2) Use of FToC correctly

(+1) Recognition of neg value on $0 \le t \le 3$

(+1) Exact value